

SENIOR STRUCTURED FINANCE OBLIGATIONS ARE NOT ECONOMIC CATASTROPHE BONDS*

Andreas Blöchlinger[†]

Zürcher Kantonalbank, University of Zurich

May 25, 2016

*The views expressed are my own and do not necessarily reflect the views of Zürcher Kantonalbank. I have nothing else to disclose in relation to this article.

[†]**Correspondence Information:** Andreas Blöchlinger, Zürcher Kantonalbank, Josefstrasse 222, CH-8010 Zurich, Switzerland; tel: +41 44 292 45 80, <mailto:andreas.bloechlinger@zkb.ch>

Senior Structured Finance Obligations Are Not Economic Catastrophe Bonds

Abstract

Senior structured finance obligations are backed by an asset pool and suffer losses only after subordinate tranches have been completely exhausted. For a given rating and fixed asset beta, the more diversified the asset pool, the higher is the price of the most senior tranche. Such bonds resemble government bonds which default only in the worst economic states. Bonds that fail only in the same states as the government bond must have the same default risk, the same systematic risk and the same price. But government bonds are close substitutes for the risk-less asset and not economic catastrophe bonds.

JEL CLASSIFICATION CODES: G11, G12

KEY WORDS: Collateralized Debt Obligation (CDO); Diversification; Leverage; Systematic Risk; Capital Asset Pricing Model (CAPM)

I. INTRODUCTION

Collateralized Debt Obligations (CDOs) were at the heart of the 2007–2008 financial market crisis. The CDO is the prototypical structured finance security and is a type of structured asset-backed security (ABS). A CDO is basically a promise to pay investors in a predefined sequence, based on the cash flows the CDO collects from the underlying pool of assets. The CDO is “sliced” into “tranches”, each CDO tranche receives the cash flow of interest and principal in sequence based on its priority. Jones (2008), in an article in *FT Alphaville*, considers structured finance to be “the single most important invention in finance, if not economics, in the past few decades.” Indeed, structured finance can improve the efficiency of resource allocation in an economy to allow for better risk sharing and help contain systemic risk by freeing up the banks’ balance sheets. In addition, investing into ABSs could be a powerful tool for central banks to increase the money supply and to stimulate credit markets. However, to reach these key goals requires first and foremost a proper understanding of the inherent economic risks of structured ABSs. I will present a model to establish a link between credit rating and price for different size and beta of the underlying asset pool as alternative to the model of Coval, Jurek, and Stafford (2009a).

The oft-cited paper of Coval, Jurek, and Stafford (2009a) in the *American Economic Review* investigates the risk and pricing implications of structured finance activities. They make the case that “losses on the most senior tranches [...] are largely confined to the worst economic states, suggesting that they should trade at significantly higher yield spreads than single-name bonds with identical credit ratings” (p. 630). According to Coval et al. such “*economic catastrophe bonds* should offer a large risk premium to compensate for their systematic risk” (p. 628). They further argue that “creating large diversified portfolios of economic assets (e.g., corporate bonds) and issuing prioritized capital structures of claims

against those pools, as is common in structured finance, emerges as a natural approach to manufacturing the cheapest security within a given credit rating category” (p. 657).

However, is a sovereign bond backed by the (taxable) assets of an economy not the natural equivalent of a senior structured finance obligation? Are senior CDO tranches not meant to replicate US treasuries to meet the demand for highly rated bonds? Should a sovereign bond based on a large and well diversified economy (such as the USA) really be cheaper than the sovereign bond based on a specialized economy (such as Saudia Arabia), *ceteris paribus* (i.a., the same default likelihood and the same CAPM beta of their portfolios of economic assets)? If we think this through it would mean that the sovereign bond, which represents a senior CDO tranche backed by taxable collateral, must be cheaper than the bond of a corporation, which represents a very specialized economy (again under the assumption that both have the same rating and asset beta). In my humble view, the hypotheses of Coval, Jurek, and Stafford (2009a) fly in the face of logic: In the worst economic states even “safe haven” securities such as Germany’s triple-A rated government bonds default. Securities that fail only under the same severe economic conditions as the government bond must also have the same default risk, the same systematic risk and therefore exactly the same price. But government bonds are usually considered a proxy for the risk-free asset and not economic catastrophe bonds. I will provide a rigorous proof.

The beta according to the “Nobel prize-winning” capital asset pricing model (CAPM) of Treynor (1962), Sharpe (1964), Lintner (1965), and Mossin (1966) derived from the mean-variance criterion of Markowitz (1952) reflects the systematic risk of an asset with respect to the market portfolio. The implication of the CAPM is that the covariation between individual asset returns and the market return defines the systematic risk and therefore matters for pricing. The remaining risk is assumed to be idiosyncratic, can be diversified away and

commands no premium. It must be noted that covariance is a linear risk measure and of limited suitability for measuring dependence (see, e.g., Embrechts, McNeil, and Straumann (1999)). Nonetheless, in this paper I focus on covariance risk and I neglect higher co-moments to make a straightforward and transparent comparison with Coval, Jurek, and Stafford (2009a) who “develop a simple state-contingent pricing framework [i]n the spirit of the William F. Sharpe (1964) and John Lintner (1965) CAPM” (p. 629). However, the pivotal conclusion in this paper remains true for more complex models with non-linear dependencies.¹

Given the same default likelihood (and therefore the same credit rating) and the same asset beta of the collateral pool, I will demonstrate that senior bonds based on a more diversified asset pool must trade, everything else equal, at a lower not higher yield spread compared to a less diversified collateral pool. Both the expected loss given default and the beta per unit at risk decrease with a more diversified collateral pool. Since the asset beta of the collateral pool and the default risk of each tranche are held constant by construction and because the beta of senior debt decreases due to diversification, the beta of the subordinated tranches must necessarily increase. That is, the systematic risk of senior debt falls whereas the systematic risk of the leveraged tranches rise. By diversifying the asset pool, senior tranches can be regarded as a continuous bridgeover between single-name bond with large systematic risk and risk-less asset. Furthermore, for any given transaction structure, the systematic risk of equity is higher than the beta of mezzanine which is in turn higher than the beta of senior debt. Thus, junior not senior tranches are economic catastrophe bonds! To

¹Blöchliger (2015) demonstrates that typical credit instruments show significant coskewness and cokurtosis risk which must be priced under the concept of standard risk aversion according to Kimball (1993). Covariance, coskewness and cokurtosis of senior debt in a CDO transaction all decrease with a more diversified asset pool resulting in a higher 4-moment CAPM price for the most senior tranche.

summarize, I will demonstrate that leverage and not diversification drives systematic risk.

I will focus exclusively on the theoretical model of Coval, Jurek, and Stafford (2009a), but I will remain silent on the empirical part of Coval et al. The models of Collin-Dufresne, Goldstein, and Yang (2012), Li and Zhao (2012) already offer a convincing resolution to the empirical puzzle reported by Coval et al. I will proceed as follows: Section II establishes an economic setting in the spirit of Coval, Jurek, and Stafford (2009a). Section III defines default risk, credit rating and systematic risk to avoid any ambiguity and to comply fully with current rating practice. Section IV derives the CAPM prices of equity, mezzanine and senior tranches and shows how these prices change with a more diversified collateral pool. Section V discusses the crucial differences to Coval, Jurek, and Stafford (2009a) and demonstrates that the most senior CDO tranche can be structured to be a close substitute for the risk-free bond. Section VI concludes.

II. THE ECONOMIC SETTING

The structuring of collateralized debt obligations (CDOs) proceeds in two basic steps. In the first step, a number of assets (e.g., mortgages, loans, bonds, CDSs) are pooled in a special purpose entity. In the second step, the cash flows of the underlying assets are redistributed, or tranced, across a series of financial derivatives. The priority observed in redistributing cash flows among the derivative securities, called tranches, allows some of them to have lower other higher expected payoffs than the mean cash flow of the underlying asset pool. I will consider a homogenous pool of K assets, each asset with a notional value of one.² The asset pool can also be interpreted as the market portfolio of economic assets of an economy.

²I will remain silent about the unit. But the unit can be thought of a US dollar, a Swiss franc, a good like a potato, or a representative basket of goods and services.

I have a one-period economy with the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ and I want to characterize CDO prices with different priorities/seniorities – equity, mezzanine, and senior debt – derived from an underlying asset pool consisting of a subset of assets from the market portfolio. The probability distribution of the K end-of-period cash-flows $\{Y_1, \dots, Y_K\}$ are assumed to be given by a multivariate Gaussian distribution:

$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_K \end{pmatrix} \sim N \left(\begin{pmatrix} \mu \\ \mu \\ \vdots \\ \mu \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{pmatrix} \right), \quad (1)$$

with $0 < \rho < 1$ and $0 < \mu < 1$. I assume that $0 < \mu < 1$ since I interpret each asset with an underlying notional amount of one. The expected payoff μ is smaller than the promised end-of-period notional amount due to credit risk but greater than zero. Further, I define the portfolio P_n with n underlying assets, $1 \leq n \leq K$, and the market portfolio M consisting of all K assets:

$$P_n = \frac{1}{n} \sum_{i=1}^n Y_i, \text{ and } M = \frac{1}{K} \sum_{k=1}^K Y_k,$$

so that P_n and M have both an (averaged) notional amount of one. By linear transformation, the end-of-period payoffs follow a bivariate Gaussian distribution:

$$\begin{pmatrix} M \\ P_n \end{pmatrix} \sim N \left(\begin{pmatrix} \mu \\ \mu \end{pmatrix}, \begin{pmatrix} \sigma_M^2 & \sigma_M^2 \\ \sigma_M^2 & \sigma_{P_n}^2 \end{pmatrix} \right),$$

where

$$\sigma_M^2 = \frac{1}{K}\sigma^2 + \frac{K-1}{K}\rho\sigma^2, \text{ and } \sigma_{P_n}^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2, \text{ with } \sigma_M \leq \sigma_{P_n} < \sigma_{P_{n-1}}.$$

The correlation coefficient between P_n and M is therefore given by σ_M/σ_{P_n} and for an increasing number of assets K (n) the variance of M (P_n) tends towards $\rho\sigma^2$. The linear

projection coefficient β by projecting P_n onto M is equal to one:

$$(P_n - \mu) = \beta (M - \mu_M) + \varepsilon, \text{ where } \mathbb{C}\text{OV}[\varepsilon, M] = 0 \text{ and } \varepsilon \sim N(0, \sigma_{P_n}^2 - \beta^2 \sigma_M^2),$$

since $\beta = \mathbb{C}\text{OV}[P_n, M] / \sigma_M^2 = 1$ for any $n \in \{1, \dots, K\}$. The projection coefficient β is the asset beta and is relevant for pricing in a CAPM framework:

PROPOSITION 1 (Two-moment CAPM pricing): *The CAPM price q_X of a financial derivative with \mathcal{F} -measurable random payoff X is given by:*

$$\frac{q_X}{q_0} = \mu_X - \beta_X \lambda, \quad (2)$$

where $\mu_X = \mathbb{E}[X]$, $\beta_X = \mathbb{C}\text{OV}[X, M] / \mathbb{V}[M]$ with $M = 1/K \sum_{k=1}^K Y_k$, $\lambda > 0$ is the market premium for covariance risk and q_0 is the price of the risk-free bond with a non-random end-of-period payoff Y_0 expressed as a fraction of the notional value.

The proofs can be found in the Appendix. Equation (2) is called a two-moment CAPM since only two statistical moments of X , namely mean μ_X and scaled variance contribution β_X , are pricing relevant. The payoff X is a financial derivative in the sense that X does not change the payoff of the market portfolio M . The physical risk statistics μ_X and β_X of X are independent from any preference assumptions, λ , however, is the market price of risk and depends on risk aversion. If X is the payoff of a credit derivative then the rating information alone is insufficient for pricing since the rating is silent on β_X . Due to the assumption of identically distributed payoffs in (1), the CAPM price q_P of the portfolio P_n is given by:

$$\frac{q_P}{q_0} = \mu - \beta \lambda.$$

That is, the price q_P does not depend on the number of underlying assets n . Note, q_P is expressed as a fraction of the notional value of the underlying assets, and that is the way I am going to express prices – always as a fraction of the underlying notional. This convention gives me comparable risk statistics per unit invested or per unit at risk, respectively.

III. DEFINITION OF DEFAULT RISK AND SYSTEMATIC RISK

As in Coval, Jurek, and Stafford (2009a), I am “considering a series of tranches with a fixed unconditional probability of default” (p. 661). This assumption is not restrictive: I will show that an equivalent assumption is to keep the expected loss rates constant except for the most junior and the most senior tranche. Since the most junior tranche is usually unrated and the most senior tranche typically triple-A rated, it is therefore almost irrelevant from an information content perspective whether credit ratings for CDO tranches are interpreted as ordinal assessments about expected loss rates (Moody’s) or about default probabilities (S&P, Fitch).

There is a broad consistency and uniformity across the financial industry that default is characterized as a “missed payment” as noted by the International Swaps and Derivatives Association (ISDA). This well established definition of a default event is crucial for the interpretation of credit ratings. As a consequence, to be fully consistent with best rating practice and current literature, default risk is best quantified by the probability of default as proposed by Coval, Jurek, and Stafford (2009a):

DEFINITION 1 (Default risk): Two assets have the same default risk if and only if they have the same probability of full repayment of principal amount and interest.

A credit rating is an ordinal assessment about default risk. Thus, I also introduce the following almost redundant definition to make the purpose of credit ratings crystal clear:

DEFINITION 2 (Credit rating): Two assets have the same credit rating if and only if they have the same default risk.

A reader of this paper has suggested to invest into a well diversified corporate bond portfolio instead of a single senior CDO tranche. Assuming that both assets have the same

expected loss rate and that corporate defaults are not perfectly correlated, i.e., $\rho < 1$, then the corporate bond portfolio can have the lower variance (as well as the lower CAPM beta) and is therefore less risky from a mean-variance perspective than the senior CDO tranche. However, due to diversification the probability of at least one default event and therefore the chance of a “missed payment” in the bond portfolio tends to one. The credit rating of the whole corporate bond portfolio must then be extremely low even though the rating of each single-name bond in the portfolio could be high. Hence, two assets with the same expected loss rate do not in general have the same credit rating.

Since I work with Gaussian distributed cash flows in (1), the systematic risk can be completely quantified by the CAPM beta in the spirit of Sharpe (1964):

DEFINITION 3 (Systematic risk): *Two assets have the same systematic risk if and only if they have the same CAPM beta.*

Having defined default risk, credit rating and systematic risk in the most standard way to avoid any ambiguity and to comply with best practice, I can continue with structuring credit derivatives and derive the prices for equity, mezzanine and senior debt tranches.

IV. PRICING OF TRANCHES

I am structuring derivative securities with different default risk from the underlying collateral pool P_n . I create an equity tranche, a mezzanine, and a senior debt tranche:

$$\underbrace{P_n}_{\text{asset pool}} = \underbrace{\mathbf{1}_{\{P_n \geq N_0 + N_1\}} (P_n - N_0 - N_1)}_{\text{equity}} + \underbrace{\mathbf{1}_{\{N_0 \leq P_n \leq N_0 + N_1\}} (P_n - N_0) + \mathbf{1}_{\{P_n \geq N_0 + N_1\}} N_1}_{\text{mezzanine}} + \underbrace{N_0 - \mathbf{1}_{\{P_n < N_0\}} (N_0 - P_n)}_{\text{senior debt}}, \quad (3)$$

where $N_0 > 0$ is the notional amount of outstanding senior debt and $N_1 > 0$ the par amount of outstanding mezzanine claims. According to Modigliani and Miller (1958), the value of all three liability claims must equal the asset value independent of the notional amounts N_0 and N_1 . In the following, I will keep the price of the asset pool constant for any n . Thus, if the price of any tranche rises due to n then the portfolio price of the remaining tranches must fall. Further, equity can be written as the payoff of a long call, $\max\{P_n - N_0 - N_1, 0\}$, the payoff of the mezzanine tranche as the payoff of a risk-free bond plus a short put and a long put, $N_1 - \max\{N_0 + N_1 - P_n, 0\} + \max\{N_0 - P_n, 0\}$, and the payoff of the senior debt tranche as the payout of a risk-free bond plus a short put, $N_0 - \max\{N_0 - P_n, 0\}$. It is crucial to note that mezzanine and equity securities are – unlike senior debt – leveraged securities.

Due to the multivariate Gaussian distribution of cash flows, the default probabilities or hitting probabilities of the mezzanine and senior debt tranche are given by $\Phi(-DD_1)$ and $\Phi(-DD_0)$, where DD_0 and DD_1 are the so-called distances to default:

$$DD_0 = \frac{\mu - N_0}{\sigma_{P_n}}, \text{ and } DD_1 = \frac{\mu - N_0 - N_1}{\sigma_{P_n}}.$$

The higher the number of underlying assets n , the lower is the volatility σ_{P_n} .

As in Coval, Jurek, and Stafford (2009a) and mentioned above, I will keep the “credit rating - as proxied by the unconditional default probability - of the securities under consideration fixed” (p. 631). Hence, when I fix the distances to default for any n , I automatically fix the default probability of the mezzanine and senior tranche and I can write the notional amounts N_0 , N_1 as a function of σ_{P_n} :

$$N_0 = \mu - DD_0\sigma_{P_n}, \quad N_1 = \sigma_{P_n}(DD_0 - DD_1), \quad \text{such that } N_0 + N_1 = \mu - DD_1\sigma_{P_n}. \quad (4)$$

To obtain the CAPM value of the three structured asset backed securities – equity, mezzanine,

senior debt – I first need to derive the mean payoff μ_X and then the variance contribution β_X for quantifying the systematic risk:

PROPOSITION 2 (Expected payoff μ_X): *For fixed default probabilities, $DD_0 > DD_1 > 0$, the mean payoffs per unit notional with n underlying assets are given by:*

$$\begin{aligned}\mu_{equity,n} &= \frac{\sigma_{P_n} [\phi(DD_1) + DD_1 \Phi(DD_1)]}{1 - \mu + DD_1 \sigma_{P_n}} < \mu_{equity,n-1} \\ \mu_{mezz,n} &= DD_0 \frac{\Phi(-DD_1) - \Phi(-DD_0)}{DD_0 - DD_1} - \frac{\phi(DD_1) - \phi(DD_0)}{DD_0 - DD_1} + \Phi(DD_1) = \mu_{mezz,n-1} \\ \mu_{debt,n} &= \frac{\mu \Phi(-DD_0) - \sigma_{P_n} \phi(DD_0)}{\mu - DD_0 \sigma_{P_n}} + \Phi(DD_0) > \mu_{debt,n-1},\end{aligned}$$

where $\Phi(\cdot)$ is the cdf and $\phi(\cdot)$ the pdf of a standard Gaussian variable. The expected payoff of the debt tranche (as a fraction of notional value) increases with an increasing number of underlying assets n . The expected payoff of the mezzanine tranche is unaffected by n . The expected payoff of the equity tranche decreases with increasing n .

Apart from the most junior and the most senior tranche, by fixing default probabilities, the expected loss rate of any tranche in between is fixed as well. Thus:

COROLLARY 1: *Except for the most junior and the most senior tranches, two CDO tranches have the same credit rating if and only if they have the same expected loss rate.*

But diversification reduces the expected loss rate of senior debt even when default probabilities and the expected loss rate of mezzanine tranches are held constant. However, since the most senior tranche is typically triple-A rated and the first-loss tranche unrated, a CDO credit rating can be equivalently interpreted as an ordinal assessment about the default probability or the expected loss rate. Next, I turn to the CAPM beta:

PROPOSITION 3 (Systematic risk β_X): *For fixed default probabilities, $DD_0 > DD_1 > 0$, and given asset beta β , the betas of equity, mezzanine and senior debt tranche per unit*

notional with n underlying assets are given by:

$$\begin{aligned}\beta_{equity,n} &= \beta \frac{\Phi(DD_1)}{1 - \mu + \sigma_{P_n} DD_1} > \beta_{equity,n-1} \\ \beta_{mezz,n} &= \beta \frac{\Phi(-DD_1) - \Phi(-DD_0)}{\sigma_{P_n} (DD_0 - DD_1)} > \beta_{mezz,n-1} \\ \beta_{debt,n} &= \beta \frac{\Phi(-DD_0)}{\mu - DD_0 \sigma_{P_n}} < \beta_{debt,n-1}.\end{aligned}$$

The beta per unit notional of senior debt decreases with an increasing number of underlying assets n . The beta per unit notional of a mezzanine tranche increases with increasing n . The beta per unit notional of equity increases with increasing n .

Note, when the senior debt beta decreases, the beta of the remaining tranches must increase, since the price of the asset pool and therefore the asset beta β is held constant by construction. If I further assume a CAPM relation, I can use these two risk statistics for pricing senior debt, mezzanine and equity securities, i.e., the price of a security $g(\mu_X, \beta_X)$ is given as a function of μ_X and β_X . Thus, I can combine Proposition 2 and Proposition 3 to establish the following proposition regarding CAPM pricing of tranches:

PROPOSITION 4 (CAPM price $g(\mu_X, \beta_X)$): *For fixed default probabilities, $DD_0 > DD_1 > 0$, and given asset beta β , the CAPM prices per unit notional and n underlying assets of equity, mezzanine and senior debt are given by:*

$$\begin{aligned}\frac{q_{equity,n}}{q_0} &= \mu_{equity,n} - \beta_{equity,n} \lambda < \frac{q_{equity,n-1}}{q_0} \\ \frac{q_{mezz,n}}{q_0} &= \mu_{mezz,n} - \beta_{mezz,n} \lambda < \frac{q_{mezz,n-1}}{q_0} \\ \frac{q_{debt,n}}{q_0} &= \mu_{debt,n} - \beta_{debt,n} \lambda > \frac{q_{debt,n-1}}{q_0},\end{aligned}$$

where λ is the unit price of covariance risk with the market portfolio M , q_0 is the unit price of the risk-less bond, the other variables are given in Proposition 2 and Proposition

3. *The CAPM price of senior debt increases with increasing number of underlying assets n , the CAPM price of mezzanine decreases with increasing n and the CAPM price of equity decreases with an increasing number of underlying assets n .*

V. DISCUSSION

Let me discuss two important results in the following: First, I discuss the crucial differences to Coval, Jurek, and Stafford (2009a). Second, I demonstrate how to structure an almost risk-free asset that resembles the government bond.

V.A. Differences to Coval et al.

Coval, Jurek, and Stafford (2009a) “show that losses on the most senior tranches referencing an index of investment grade credit default swaps are largely confined to the worst economic states, suggesting that they should trade at significantly higher yield spreads than single-name bonds with identical credit ratings. Surprisingly, this implication turns out not to be supported by the data” (p. 630). Proposition 4 contradicts this statement since $q_{\text{debt},n} > q_{\text{debt},n-1}$ for any $n \in \{2, \dots, K\}$, where $q_{\text{debt},1}$ is the single-name bond. Senior debt has the lowest CAPM price when $n = 1$ and the highest CAPM price when the underlying asset pool is the market portfolio M , i.e., $n = K$. Thus, since their proposition is wrong to begin with, it is hardly surprising that it is not supported by the data.

But I also reject with Proposition 4 the hypothesis of Coval, Jurek, and Stafford (2009a) regarding the equity tranche: “In other words, because the equity tranche bears the first losses on the underlying portfolio, it is exposed primarily to diversifiable, idiosyncratic losses. The benign nature of the underlying risk [...] stands in marked contrast to the tranche’s

popular characterization as “toxic waste” ” (p. 660). This interpretation is wrong since the CAPM equilibrium price of equity falls the more diversified the collateral pool, i.e., $q_{\text{equity},n} < q_{\text{equity},n-1}$ for any $n \in \{2, \dots, K\}$. In case of $K = n$ there is anyway no diversifiable, idiosyncratic risk left, the underlying asset pool is the market portfolio. Thus, the equity tranche has the lowest CAPM price and is therefore “most toxic” when the underlying collateral pool is M . Junior not senior bonds are economic catastrophe bonds!

The problem with the conclusions of Coval, Jurek, and Stafford (2009a) is that they analyze the risk locally but interpret it globally. They show that by fixing default probabilities, the so-called risk-neutral default probabilities increase with increasing n . I can replicate their finding within my setting by calculating the price of a digital option or a digital tranche that pays out one unit if the portfolio payoff P_n is greater than N_0 or N_1 and zero else:

PROPOSITION 5 (Proposition of Coval, Jurek, and Stafford (2009a)): *For fixed physical default probabilities, $DD_0 > DD_1 > 0$, and given asset beta β , the CAPM prices of digital tranches are given by:*

$$\frac{q_{\text{digital mezz},n}}{q_0} = \Phi(DD_1) - \lambda \frac{\beta}{\sigma_{P_n}} \phi(DD_1) < \frac{q_{\text{digital mezz},n-1}}{q_0}$$

$$\frac{q_{\text{digital debt},n}}{q_0} = \Phi(DD_0) - \lambda \frac{\beta}{\sigma_{P_n}} \phi(DD_0) < \frac{q_{\text{digital debt},n-1}}{q_0},$$

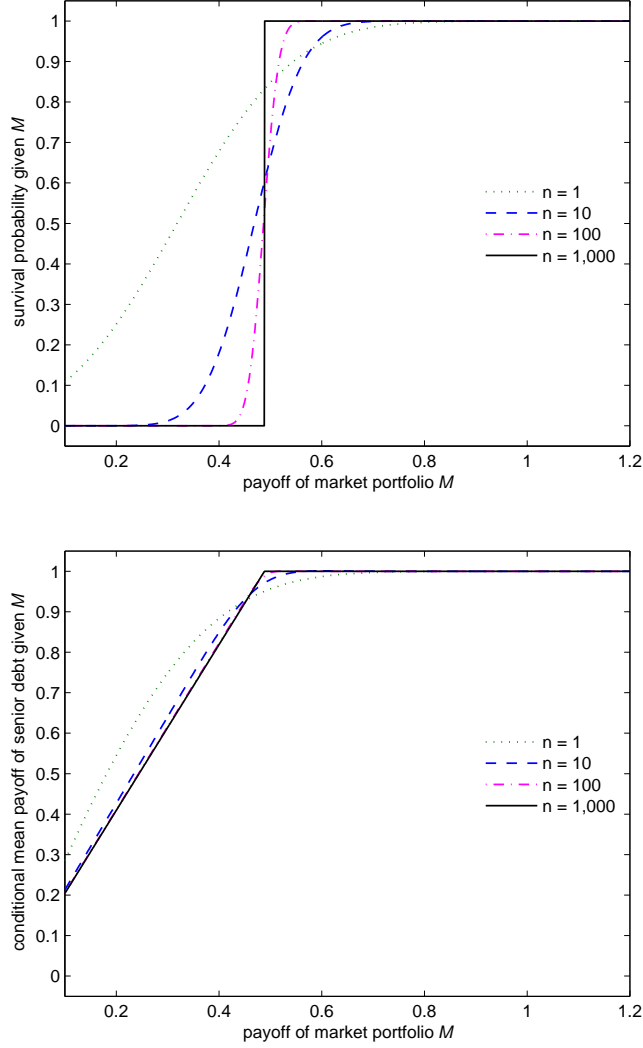
such that

$$\beta_{\text{digital mezz},n} = \frac{\beta}{\sigma_{P_n}} \phi(DD_1) > \frac{\beta}{\sigma_{P_n}} \phi(DD_0) = \beta_{\text{digital debt},n}.$$

The prices of digital tranches decrease with an increasing number of underlying assets n .

The digital mezzanine tranche has always the lower price than the digital senior tranche. But importantly, the price of a digital tranche is not the price of the whole tranche but simply the price of a binary bet against low values of P_n (and this bet does not change the

FIGURE I: State Contingent Payoff of Digital Tranche and Senior Debt



I replicate Figure 1 and 2 in Coval, Jurek, and Stafford (2009a) within a two-moment CAPM. The state of the economy is described by the market portfolio $M \sim N(0.9, 0.1769^2)$ with $K = 1,000$ assets. For any n , all digital tranches and senior debt securities have a default probability of 1% and the asset pool P_n has a mean of 0.9 and variance $\sigma_{P_n}^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2$ with $\rho = 0.5$, $\sigma = 0.25$. Unlike digital tranches (upper panel), senior debt securities (lower panel) have significant recovery values in bad states, i.e., for realization of M smaller than 0.4886 (default with $n = 1,000$).

composition of the market portfolio). The higher n the more this lottery corresponds to a digital gamble against bad states, in particular when the asset pool is the market portfolio, i.e., $n = K$, as illustrated in the upper panel of Figure I. By definition, a digital tranche has no recovery value in case of default, but senior debt has a recovery value as illustrated in the lower panel of Figure I.

In effect, Coval, Jurek, and Stafford (2009a) make only a marginal or local consideration. That is, they only consider the beta of the first unit one potentially loses by investing into a certain tranche, but to obtain the beta of the whole tranche, I have to average over all units at risk. That is, I have to make a global consideration:

$$\frac{1}{N} \int_a^b \underbrace{\frac{\beta}{\sigma_n} \phi\left(-\frac{\mu - \xi}{\sigma_{P_n}}\right)}_{\text{beta of digital tranche}} d\xi = \frac{1}{N} \beta \underbrace{\left[\Phi\left(-\frac{\mu - b}{\sigma_{P_n}}\right) - \Phi\left(-\frac{\mu - a}{\sigma_{P_n}}\right) \right]}_{\text{average beta of series of digital tranches}}, \quad (5)$$

where N is the notional amount, a and b are the attachment and detachment point. If I impute $a = N_0$, $b = N_0 + N_1$, $N = N_1$ into Equation 5, I have an alternative derivation of the beta of the mezzanine tranche $\beta_{\text{mezz},n}$ from Proposition 3 in the form of an average of digital betas. Similarly, with $a = N_0 + N_1$, $b = \infty$, $N = 1 - N_0 - N_1$ I obtain the equity beta $\beta_{\text{equity},n}$, and the senior debt beta $\beta_{\text{debt},n}$ is obtained with $a = -\infty$, $b = N_0$, $N = N_0$.

Unlike a single digital tranche, senior debt consists of a series of digital tranches and both the expected loss given default and the averaged beta decrease with an increasing asset pool resulting in a higher price. This finding is economically relevant since senior debt is normally the largest funding source. The most senior tranche in a structured ABS transaction is often called super senior tranche and is by construction unleveraged and essentially the largest slice of a CDO. Even in the empirical analysis of Coval, Jurek, and Stafford (2009a) regarding CDX tranches, the most senior tranche covers a majority of 70% of the underlying notional amount in the asset pool. So, I want to rectify the central conclusion of Coval et al.

“Consequently, despite having an unaltered credit rating, the [most senior] securities in the sequence offer progressively ~~less~~ [more] protection against economic catastrophe. In order to bear this ~~increased~~ [decreased] level of systematic risk, the marginal investor demands ~~additional~~ [less] compensation, causing the tranche price [per unit notional] to ~~fall~~ [rise].”

Coval, Jurek, and Stafford (2009a), (p. 633)

The original statement holds true for all leveraged securities. However, Coval et al. interpret their finding for the most senior tranche:

“Investors in senior tranches of collateralized debt obligations bear enormous systematic risk. [...] The key to understanding the market’s dramatic rise and fall is to recognize [...] their ability to concentrate systematic risks in the most senior tranches.”

Coval, Jurek, and Stafford (2009b), (p. 18/19)

V.B. A Close Substitute for the Risk-less Asset

As derived above, the systematic risk of a mezzanine tranche is higher than that of a senior tranche, all the more the more diversified the collateral pool. The systematic risk can therefore not be concentrated in the most senior tranches. This theoretical finding is also very intuitive, in every bad state of the economy that the senior debt tranche is hit, the mezzanine tranche is wiped out as well but not the other way around. The more diversified the underlying collateral pool the better is the protection but the higher is the price. To boot, from Proposition 4 you can immediately derive that a debt security with significant over-collateralization (similar to a covered bond or repo) resembles the default-free bond:

COROLLARY 2 (Structuring a close substitute for the risk-free asset): *The lower the default probability, $DD_0 \rightarrow \infty$, the closer is the price of the senior debt tranche to the price of the risk-free asset, $q_{debt,n-1} > q_{debt,n} \rightarrow q_0$ for any $n \in \{2, \dots, K\}$.*

Hence, the senior tranche based on K assets (market portfolio) and long distance to default DD_0 fails to deliver in the worst states (huge market losses), yet its price $q_{debt,K}$ is nonetheless close to the price q_0 of the risk-free bond (and closer than the price of a single-name bond $q_{debt,1}$ with the same distance to default DD_0). Thus, the following statement of Coval, Jurek, and Stafford (2009b) is in my view only half-correct: “Securities that fail to deliver their promised payments in the “worst” economic states will have low values, because these are precisely the states where a dollar is most valuable” (p. 628). Yes, senior debt defaults in the worst states but *if and only if* mezzanine and equity have experienced a complete loss. Only the risk-free bond offers better insurance against the worst states at a marginally higher price. But after all, the risk-free asset is also a scarce resource. In my model there is not even an explicit risk-free asset, one has to structure an approximative risk-free security via Proposition 2, $q_{debt,K}$ is de facto the price of the risk-free asset, $q_0 \approx q_{debt,K} > q_{debt,1}$. I mean, what, exactly, would they propose to hold instead? They could invest into a single-name bond and buy protection, but this insurance is also exposed to systematic risk through counterparty risk from the insurer. But in the worst economic states even “safe haven” counterparties such as Switzerland or the US might fail.

VI. CONCLUSION

This paper demonstrates that the most senior bond of a structured finance transaction offers better protection against bad states of the economy relative to equally rated single-name corporate bonds (given the same underlying asset beta). In my view, this important

observation has not been well articulated neither in the academic nor practitioner literature. On the contrary, I show that the conclusion of Coval, Jurek, and Stafford (2009b) – nota bene in one of Economic’s most prestigious outlets – about the market for structured finance obligations, “the key to understanding the market’s dramatic rise and fall is to recognize [...] their ability to concentrate systematic risks in the most senior tranches,” (p. 19) is wrong. Coval, Jurek, and Stafford (2009a) also state that “because the equity tranche bears the first losses on the underlying portfolio, it is exposed primarily to diversifiable, idiosyncratic losses” (p. 660). In this paper I prove the opposite: Junior and not senior bonds are economic catastrophe bonds.

In fact, given a constant asset beta and fixed default risk (in the sense of a constant probability of full repayment of nominal amount and interest), diversification moves the systematic risk out of the senior debt tranche into subordinated tranches. Due to diversification, the beta and the expected loss rate of the most senior tranche become so low that it can even be considered a close substitute for a government bond or the default-free bond, respectively. Because most senior tranches are virtually free of risk, they are correctly triple-A rated. However, the systematic risk of all but the most senior tranche – in other words all leveraged tranches – can be considerably higher than that of an equally rated senior (and therefore unleveraged) single-name bond. Leverage and not diversification powers systematic risk!

Since the unleveraged tranche is typically the largest slice of a structured finance transaction, it is key to make public the risk characteristics of the different tranches to the average investor in order to revive the private securitization market. Otherwise, some investors remain reluctant to invest even in low-risk securities, which in fact closely resemble government bonds, that they falsely associate now with economic catastrophe bonds caused by some misconceptions in the current literature. Finally, for central banks the investments into senior

ABSs could be a key tool to stimulate credit markets and to increase the money supply. However, such investments arguably require the broad acceptance of the general public who still largely thinks all such papers are toxic catastrophe bonds. I hope that my paper can make a modest contribution to get rid of this erroneous belief.

APPENDIX: PROOFS

Proof of Proposition 1. The positive Radon-Nikodym derivative Z ,

$$Z = 1 - \lambda \frac{M - \mu_M}{\sigma_M^2},$$

has mean one and induces the measure change from the real-world probability measure \mathbb{P} to the so-called risk-neutral martingale measure \mathbb{Q} . In finance terms, the variable Z is called the CAPM pricing kernel.³ By the fundamental theorem of asset pricing, the compounded price $p_X = q_X/q_0$ of the \mathcal{F} -measurable financial derivative X is the expected payoff under the risk-neutral martingale measure \mathbb{Q} :

$$p_X = \mathbb{E}^{\mathbb{Q}}[X] = \mathbb{E}[Z X] = \mathbb{E}[X] - \frac{\text{COV}[X, M]}{\sigma_M^2} \lambda = \mu_X - \beta_X \lambda,$$

where $\text{COV}[X, M] = \mathbb{E}[X M] - \mathbb{E}[X] \mathbb{E}[M]$ and $\mathbb{V}[M] = \text{COV}[M, M] = \sigma_M^2$ under the physical measure \mathbb{P} . □

Proof of Proposition 2. I can write the expected payoff of the asset pool as the linear combination of expected payoff per unit notional amount of equity, mezzanine and senior debt tranche:

$$\begin{aligned} \mathbb{E}[P_n] &= \mu = (1 - N_0 - N_1) \mu_{\text{equity},n} + N_1 \mu_{\text{mezz},n} + N_0 \mu_{\text{debt},n} \\ &= (1 - \mu + DD_1 \sigma_{P_n}) \mu_{\text{equity},n} + \sigma_{P_n} (DD_0 - DD_1) \mu_{\text{mezz},n} + (\mu - DD_0 \sigma_{P_n}) \mu_{\text{debt},n}, \end{aligned}$$

where the second line follows from (4). The mean payoff of the senior debt tranche can be

³In the two-moment CAPM, the Radon-Nikodym derivative $Z = 1 - \lambda(M - \mathbb{E}[M])/\mathbb{V}[M]$ induces the measure change from the physical probability \mathbb{P} to the risk-neutral measure \mathbb{Q} such that $\mathbb{Q}\{A\} = \mathbb{E}[Z \mathbf{1}_A]$ for any $A \in \mathcal{F}$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Although my Gaussian assumption allows for negative values of M (and also for negative values of Z), the likelihood is small if $\mathbb{E}[M]$ is close to one and $\mathbb{V}[M] \ll 1$.

separated into two expectation terms:

$$\begin{aligned}\mathbb{E} [\mathbf{1}_{\{P_n > N_0\}}] N_0 &= N_0 \Phi(DD_0) \\ \mathbb{E} [\mathbf{1}_{\{P_n \leq N_0\}} P_n] &= \mu \Phi(-DD_0) - \sigma_{P_n} \phi(DD_0).\end{aligned}\tag{6}$$

The second term is based on the mean of a truncated Gaussian variable. That is, the second expectation term is the mean payoff given default $\mu - \sigma_{P_n} \phi(DD_0) / \Phi(-DD_0)$ multiplied by the default probability $\Phi(-DD_0)$. The first expectation term is the mean payoff of a digital tranche, i.e., paying out N_0 in case of survival and zero else. For equity and mezzanine I can proceed equivalently, to obtain the mean payoff per unit notional:

$$\begin{aligned}\mu_{\text{equity},n} &= \frac{1}{1 - N_0 - N_1} \{ \mu - \mu \Phi(-DD_1) + \sigma_{P_n} \phi(DD_1) - (N_0 + N_1) \Phi(DD_1) \} \\ \mu_{\text{mezz},n} &= \frac{1}{N_1} \{ (\mu - N_0) [\Phi(-DD_1) - \Phi(-DD_0)] - \sigma_{P_n} [\phi(DD_1) - \phi(DD_0)] + N_1 \Phi(DD_1) \} \\ \mu_{\text{debt},n} &= \frac{1}{N_0} \{ \mu \Phi(-DD_0) - \sigma_{P_n} \phi(DD_0) + N_0 \Phi(DD_0) \}.\end{aligned}$$

Since the notional amounts N_0, N_1 can be expressed in terms of DD_0, DD_1 , and σ_{P_n} when default probabilities are fixed as derived in (4), I obtain the desired terms for $\mu_{\text{equity},n}$, $\mu_{\text{mezz},n}$, and $\mu_{\text{debt},n}$.

The mean of the mezzanine tranche $\mu_{\text{mezz},n}$ is unaffected by σ_{P_n} . By assumption $0 < \mu < 1$ and since $DD_0 \Phi(-DD_0) - \phi(DD_0) < 0$ for any $DD_0 \in \mathbb{R}$, the marginal effect of σ_{P_n} on $\mu_{\text{equity},n}$, $\mu_{\text{debt},n}$ are given as follows:

$$\begin{aligned}\frac{\partial \mu_{\text{equity},n}}{\partial \sigma_{P_n}} &= (1 - \mu) \frac{\phi(DD_1) + DD_1 \Phi(DD_1)}{[1 - \mu + DD_1 \sigma_{P_n}]^2} > 0 \Rightarrow \frac{\partial \mu_{\text{equity},n}}{\partial \sigma_{P_n}} \frac{\partial \sigma_{P_n}}{\partial n} = \frac{\partial \mu_{\text{equity},n}}{\partial n} < 0 \\ \frac{\partial \mu_{\text{debt},n}}{\partial \sigma_{P_n}} &= \mu \frac{DD_0 \Phi(-DD_0) - \phi(DD_0)}{[\mu - DD_0 \sigma_{P_n}]^2} < 0 \Rightarrow \frac{\partial \mu_{\text{debt},n}}{\partial \sigma_{P_n}} \frac{\partial \sigma_{P_n}}{\partial n} = \frac{\partial \mu_{\text{debt},n}}{\partial n} > 0.\end{aligned}$$

The mean payoff per unit notional invested into the equity (debt) tranche decreases (increases) with increasing n since $\partial \sigma_{P_n} / \partial n < 0$. The mean payoff of the mezzanine tranche is independent of n . \square

Proof of Proposition 3. The beta of the asset pool is one and can be written as the linear combination of the betas per unit notional of equity, mezzanine and senior debt tranche:

$$\begin{aligned}\beta &= (1 - N_0 - N_1) \beta_{\text{equity},n} + N_1 \beta_{\text{mezzanine},n} + N_0 \beta_{\text{debt},n} \\ &= (1 - \mu + DD_1 \sigma_{P_n}) \beta_{\text{equity},n} + \sigma_{P_n} (DD_0 - DD_1) \beta_{\text{mezz},n} + (\mu - DD_0 \sigma_{P_n}) \beta_{\text{debt},n},\end{aligned}$$

where the second line follows from (4). To obtain the beta of the equity tranche, I first calculate the covariance between the payoff $\mathbf{1}_{\{P_n \geq N_0 + N_1\}}(P_n - N_0 - N_1)$ and M :

$$\begin{aligned}\text{COV} [\mathbf{1}_{\{P_n \geq N_0 + N_1\}}(P_n - N_0 - N_1), M] &= \mathbb{E} [\mathbf{1}_{\{P_n \geq N_0 + N_1\}}(P_n - N_0 - N_1)(M - \mu_M)] \\ &= \frac{\sigma_M^2}{\sigma_{P_n}^2} \mathbb{E} [\mathbf{1}_{\{P_n \geq N_0 + N_1\}}(P_n - N_0 - N_1)(P_n - \mu)] \\ &= \sigma_M^2 \Phi(DD_1).\end{aligned}$$

The first equality applies the covariance definition. The second equality follows from a linear projection of M onto P_n with a stochastically independent residual Gaussian variable ϵ :

$$(M - \mu_M) = \frac{\sigma_M^2}{\sigma_{P_n}^2} (P_n - \mu) + \epsilon, \text{ with } \mathbb{E}[\epsilon P_n] = 0, \epsilon \sim N\left(0, \sigma_M^2 \left(1 - \frac{\sigma_M^2}{\sigma_{P_n}^2}\right)\right). \quad (7)$$

For the third equality I resort to the mean and the variance of a truncated Gaussian variable, in particular, the integral in (6) and the following definite integral of a Gaussian function:

$$\mathbb{E} [\mathbf{1}_{\{P_n \leq N_0\}} P_n^2] = (\mu^2 + \sigma_{P_n}^2) \Phi(-DD_0) - \sigma_{P_n} (\mu + N_0) \phi(DD_0).$$

Therefore,

$$\beta_{\text{equity},n} = \beta \frac{\Phi(DD_1)}{1 - N_0 - N_1} = \beta \frac{\Phi(DD_1)}{1 - \mu + \sigma_{P_n} DD_1},$$

where the second equality follows from (4). I can proceed equivalently to obtain the betas of mezzanine and senior debt per unit notional. \square

Proof of Proposition 5. I first compute the beta of a digital option that pays out one unit in case $P_n < N_0$ and zero else:

$$\begin{aligned}\beta_{\text{digital debt},n} &= \frac{\text{COV} [\mathbf{1}_{\{P_n \geq N_0\}}, M]}{\sigma_M^2} \\ &= \frac{1}{\sigma_M^2} \mathbb{E} [\mathbf{1}_{\{P_n \geq N_0\}} (M - \mu_M)] = \frac{\beta}{\sigma_{P_n}^2} \mathbb{E} [\mathbf{1}_{\{P_n \geq N_0\}} (P_n - \mu)] = \frac{\beta}{\sigma_{P_n}} \phi(DD_0).\end{aligned}$$

The third equality follows from (7), the fourth from (6), so that the price $q_{\text{digital},N_0,n}$ is given by:

$$\mathbb{Q} \{P_n \geq N_0\} = \frac{q_{\text{digital debt},n}}{q_0} = \mu_{\text{digital debt},n} - \beta_{\text{digital debt},n} \lambda = \Phi(DD_0) - \lambda \frac{\beta}{\sigma_{P_n}} \phi(DD_0).$$

Since the standard deviation σ_{P_n} is decreasing in n but the physical survival probability $\Phi(DD_0)$ remains fixed for any n , the risk-neutral default probability increases or the risk-neutral survival probability decreases, respectively. The same argument can be applied to the default probability of the mezzanine tranche with notional amount N_1 . \square

REFERENCES

- BLÖCHLINGER, A. (2015): “Credit Rating and Pricing: Poles Apart,” Working paper, Zürcher Kantonalbank, University of Zurich.
- COLLIN-DUFRESNE, P., R. S. GOLDSTEIN, AND F. YANG (2012): “On the Relative Pricing of Long-Maturity Index Options and Collateralized Debt Obligations,” *Journal of Finance*, 67(6), 1983–2014.
- COVAL, J. D., J. W. JUREK, AND E. STAFFORD (2009a): “Economic Catastrophe Bonds,” *American Economic Review*, 99(3), 628–666.
- (2009b): “The Economics of Structured Finance,” *Journal of Economic Perspectives*, 23(1), 3–25.
- EMBRECHTS, P., A. MCNEIL, AND D. STRAUMANN (1999): “Correlation And Dependence In Risk Management: Properties And Pitfalls,” in *Risk Management: Value At Risk And Beyond*, pp. 176–223. Cambridge University Press.
- JONES, S. (2008): “Synthetic CDOs: Not Saving Anything,” *Financial Times Alphaville*, December 2.
- KIMBALL, M. S. (1993): “Standard Risk Aversion,” *Econometrica*, 61(3), 589–611.
- LI, H., AND F. ZHAO (2012): “Economic Catastrophe Bonds: Inefficient Market or Inadequate Model?,” *SSRN eLibrary*.
- LINTNER, J. (1965): “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets,” *Review of Economics and Statistics*, 47, 13–37.
- MARKOWITZ, H. (1952): “Portfolio Selection,” *The Journal of Finance*, 7(1), 77–91.

- MODIGLIANI, F., AND M. H. MILLER (1958): “The Cost of Capital, Corporate Finance and the Theory of Investment,” *American Economic Review*, 48, 261–297.
- MOSSIN, J. (1966): “Equilibrium in a Capital Asset Market,” *Econometrica*, 10, 768–783.
- SHARPE, W. F. (1964): “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk,” *Journal of Finance*, 19, 425–442.
- TREYNOR, J. L. (1962): “Toward a Theory of Market Value of Risky Assets,” unpublished manuscript.